Fundamentals of Electrical Engineering

Course Code: ME 1206

Semester Offered (Fall first semester)

Course Objectives/Goals (optional):

The goals of this course are to enable students to: Knowledge of the basic principles of electrical circuits and their most important components and their impact on electrical circuits

Course Learning Outcomes:

By the end of successful completion of this course, the student will be able to:

1- Explain precisely what the fundamental circuit variables mean

2- Apply Kirchhoff's current and voltage laws, Ohm's law, and the terminal relations describing inductive and capacitive energy-storage elements to circuit problems.

3- Simplify circuits using series and parallel equivalents and using Thevenin and Norton equivalents

4- Explain the physical underpinnings of capacitance and inductance.

Course Topics:-

1- Concept of Network and circuit .
2- •Types of Elements •
3- Types of Sources •
4- Source Transformation •
5- R-L-C Parameters •

6- Voltage - Current relationships for Passive Elements
7- • Kirchhoff's Laws.

INTRODUCTION:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, of any system can be learnt by mastering the techniques of circuit theory.
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Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it to other element of the circuit.

Active elements are of two types

a) Voltage source .

b) Current source .

A Voltage source has a specified voltage across its terminals, independent of current flowing through it.

A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance. When electrical energy is supplied to a circuit element, it will respond in one and more of the following ways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Charge and Current

Most basic quantity in an electric circuit – electric charge

Charge, *e*, is an electrical property of the atomic particles of which matter consists, measured in coulomb (C)

Basic Electrical Quantities

Basic quantities: current, voltage and power.

Electric current:

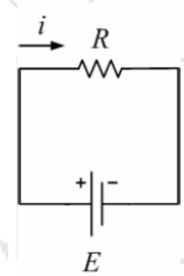
Electric current in a wire is defined as the net amount of charge that passes through the wire per unit time , and is measured in amperes (A).

$$i = \frac{dq}{dt}$$

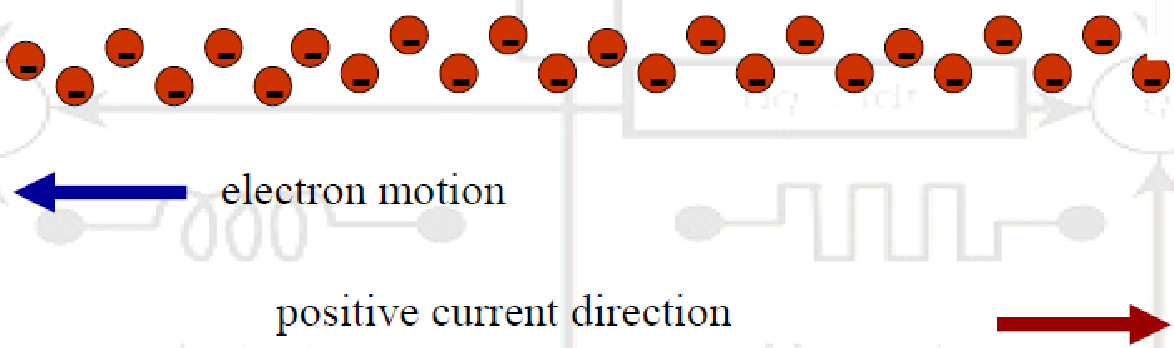
where

i = current in amperes
q = charge in coulombs
t = time in sec.

- 1 Ampere = 1 Coulomb per second (C/s)
- Current in circuits physically realized by movement of electrons.
- Direction of current must be specified by an arrow.



- By convention, current direction defined as flow of positive charge.
- Note that positive charge is not flowing physically.
- Electrons have negative charge.
- They move in the opposite direction of current.



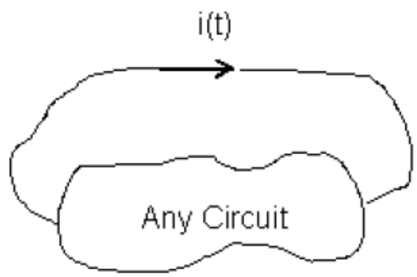
- In general, current can be an arbitrary function of time.

  Constant current is called direct current (DC).

  Current that can be represented as a sinusoidal function of time (or in some contexts a sum of sinusoids) is called alternating current (AC).

Electric circuit:

A circuit is an interconnection of electrical components.



Electric charge: is one of the fundamental quantities and exists in every atom.

Symbol: Q or q

Unit: Coulomb (C).

Electric Current: The time rate of change of charge. $i(t) = \frac{dq}{dt}$

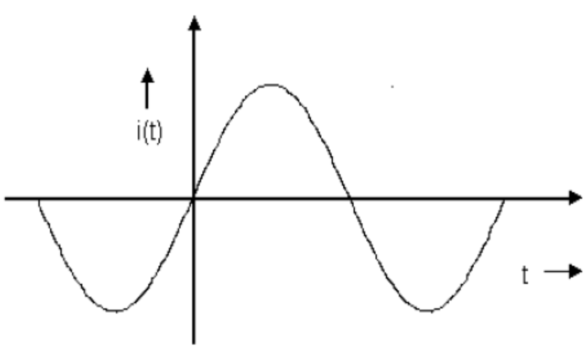
Symbol: $i(t)$ or i or I depending on whether the current is constant or time varying quantity.

Unit: Ampere (A); $1 \cdot A = \frac{1 \cdot C}{1 \cdot s}$

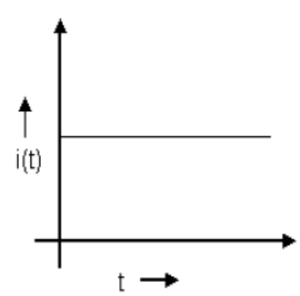
Types of currents:

→ Alternating current (ac)

→ Direct current (dc)



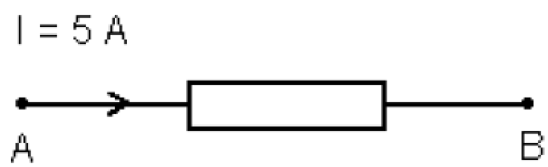
Alternating Current



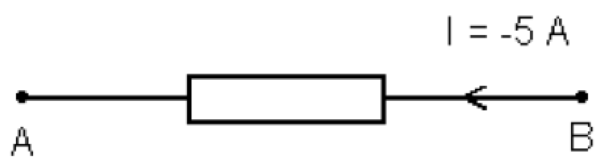
Direct Current

Current flow in a conductor (wire or any element) is specified by two indicators.

1. Direction of current flow, and
2. Value (magnitude)
   - For ac currents, the magnitude varies with time
   - For dc currents, current has a steady value



5A current flows from point A to point B; this is same as a negative current of magnitude 5A flowing from B to A.



Example 1.1

Determine the current in a circuit if a charge of 80 coulombs passes a given point in 20 seconds (s).

Solution:

$$I = \frac{Q}{t} = \frac{80}{20} = 4\ A$$

Example 1.2

How much charge is represented by 4,600 electrons?

Solution:

Each electron has - 1.602×10^{-19} C. Hence 4,600 electrons will have:

$-1.602 \times 10^{-19} \times 4600 = -7.369 \times 10^{-16}$ C

Example 1.3

The total charge entering a terminal is given by $q = 5t\sin 4\pi t\, mC$. Calculate the current at $t = 0.5\ s$.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5t\sin 4\pi t) = (5\sin 4\pi t + 20\pi t\cos 4\pi t)\ mA$$

At $t = 0.5\ s$.

$i = 31.42\ mA$

Example 1.4

Determine the total charge entering a terminal between $t = 1\ s$ and $t = 2\ s$ if the current passing the terminal is $i = (3t^2 - t)\ A$.

Solution:

$$q = \int_{t=1}^{t=2} i\,dt = \int_1^2 (3t^2 - t)\,dt = \left(t^3 - \frac{t^2}{2}\right)_1^2 = (8 - 2) - \left(1 - \frac{1}{2}\right) = 5.5C$$

Voltage

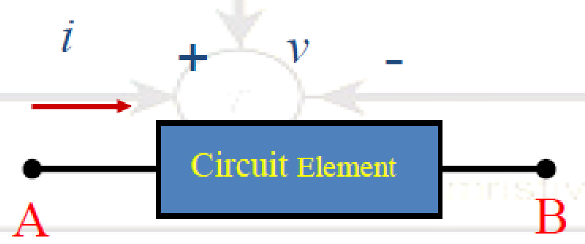
Voltage is the energy absorbed or expended as a unit charge moves from one point to the other.

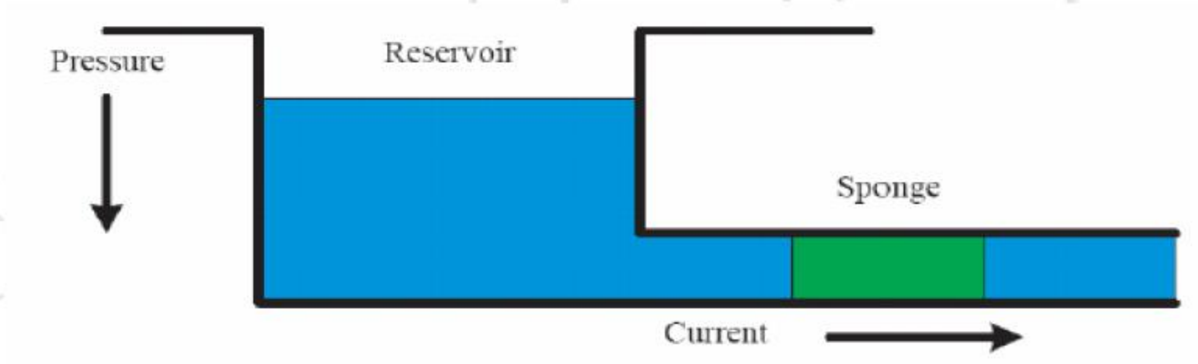
- Analogous to pressure in hydraulic system.
- Sometimes called potential difference.
- Can be created by a separation of charge.
- Is a measure of the potential between two points.
- Voltage pushes charge in one direction.

- We use polarity (+ and – on batteries) to indicates which direction the charge is being pushed
- Voltage is the energy required to move a unit charge through an element, measured in volts (V)

$$v = \frac{d\omega}{dq}$$

where v = voltage in volts
ω = energy in Joules
q = charge in coulombs





Voltage ~ Pressure

Electric Current ~ Water Current

Sponge ~ Resistance

Electrical Power

Time rate of expending or absorbing energy and is measured by Watts.

$$p = \frac{d\omega}{dt}$$

$$p = \left(\frac{d\omega}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

where p = power in watts
ω = energy in Joules
t = time in seconds
q = charge in coulombs
i = current in apperes
v = voltage in volts

By convention

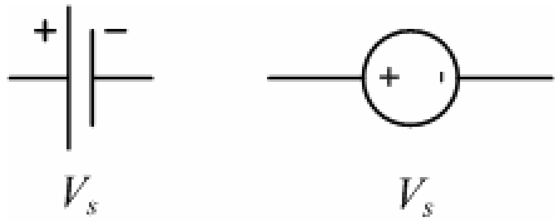
– Circuit elements that absorb power have a positive value of p.
– Circuit elements that produce power have a negative value of p.

Elements of electrical circuits

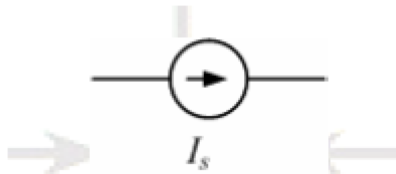
Active elements

Active elements are the elements that can generate energy or power, such as
voltage and current sources.

- Ideally, a voltage source produces *Vs* volts regardless of the current absorbed or produced by the connected device.



- Ideally, a current source produces *Is* amps regardless of the current in the connected device.



- In a particular circuit, there can be active elements that absorb power – for example, a battery being charged.

Passive elements

passive elements are the elements that can not generate energy, such as resistors, capacitors and inductors.

resistors

- The ability of a material to resist (impede, obstruct) the flow charge is called its resistivity. It is represented by the letter R.
- A resistor is a circuit element that dissipates electrical energy (usually as heat)
- Real-world devices that are modeled by resistors: incandescent light bulbs, heating elements, long wires
- Resistance is measured in Ohms (Ω)
- Resistor is indicated by the symbol 

- Resistance of a wire depends on some factors like as length (L), cross-sectional area (A) and resistivity of material (ρ).

$$R = \frac{\rho L}{A}$$

Where ρ resistivity in Ω.m
L length in m
A cross-section area in m^2

- The conductance (G) of a pure resistor is the reciprocal of its resistance. The unit of conductance is the siemens (S) or mho (℧).

$$G = \frac{1}{R}$$

Ohm's Law

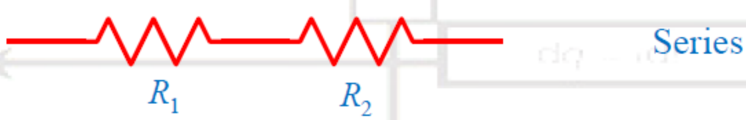
Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them.

➤The mathematical equation that describes this relationship is:

$$i = \frac{v}{R}$$

where v is the potential difference measured across the resistance in units of volts; i is the current through the resistance in units of amperes and R is the resistance of the conductor in units of ohms.

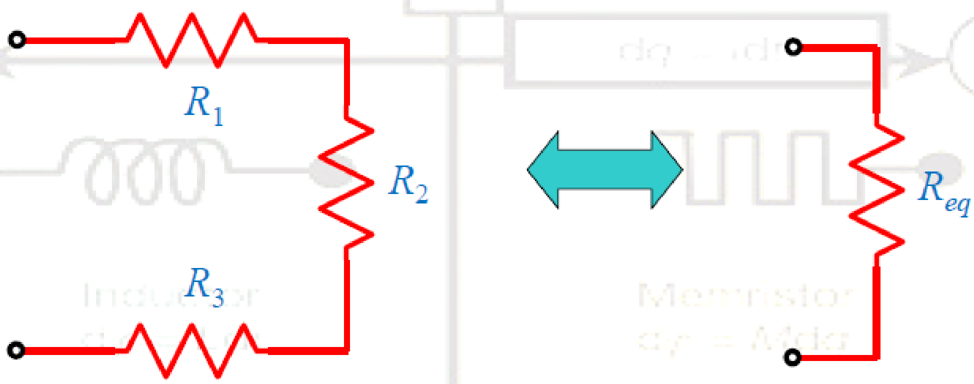
Two elements are in series if the current that flows through one must also flow through the other.



➤ If we wish to replace the two series resistors with a single equivalent resistor whose voltage-current relationship is the same, the *equivalent* resistor has a value given by

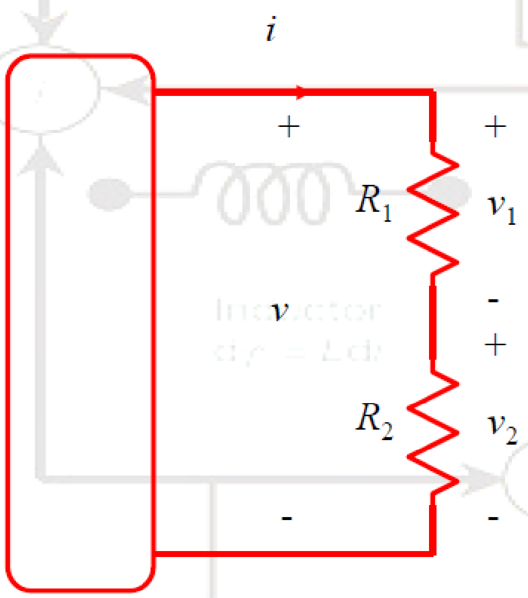
$$R_{eq} = R_1 + R_2$$

➢ For N resistors in series, the equivalent resistor has a value given by:



$$R_{eq} = R_1 + R_2 + R_3 + \Lambda\ + R_N$$

➢ Consider two resistors in series with a voltage v across them:



Voltage division:

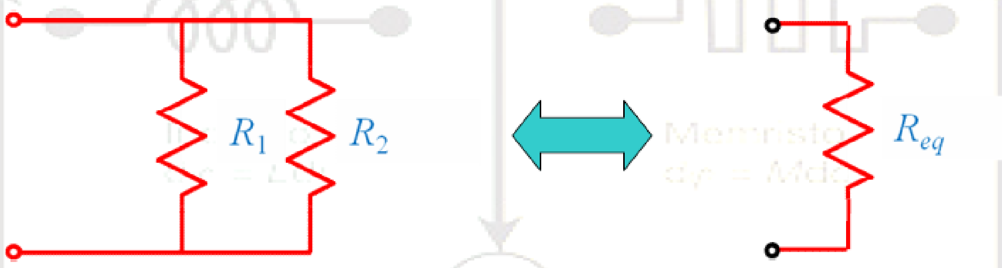
$$v_1 = v\frac{R_1}{R_1 + R_2}$$

$$v_2 = v\frac{R_2}{R_1 + R_2}$$

Resistors in Parallel

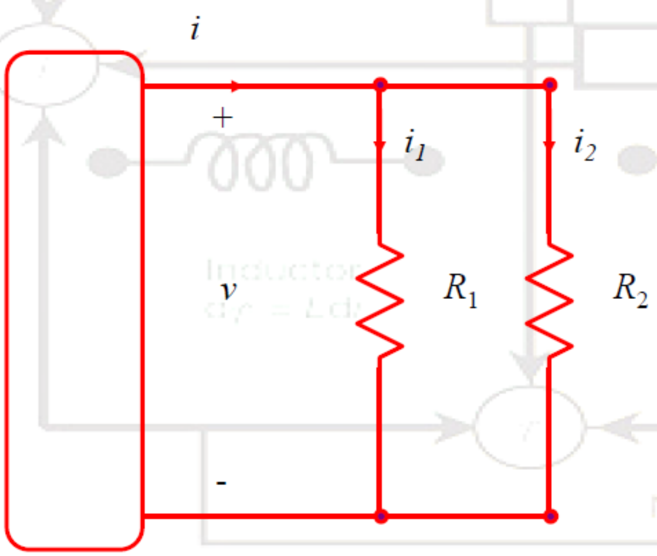
- When the terminals of two or more circuit elements are connected to the same two nodes, the circuit elements are said to be in parallel.

- If we wish to replace the two parallel resistors with a single equivalent resistor whose voltage-current relationship is the same, the *equivalent* resistor has a value given by



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- Consider two resistors in parallel with a voltage v across them:



Current division:

$$i_1 = i \frac{R_2}{R_1 + R_2}$$

$$i_2 = i \frac{R_1}{R_1 + R_2}$$

Kirchhoff's Circuit Laws

Kirchhoff's Laws

What are Kirchhoff's Laws?

- Kirchhoff's laws govern the conservation of charge and energy in electrical circuits.

•Kirchhoff's Laws

1.The junction rule

2.The closed loop rule

Kirchhoff's currents Laws

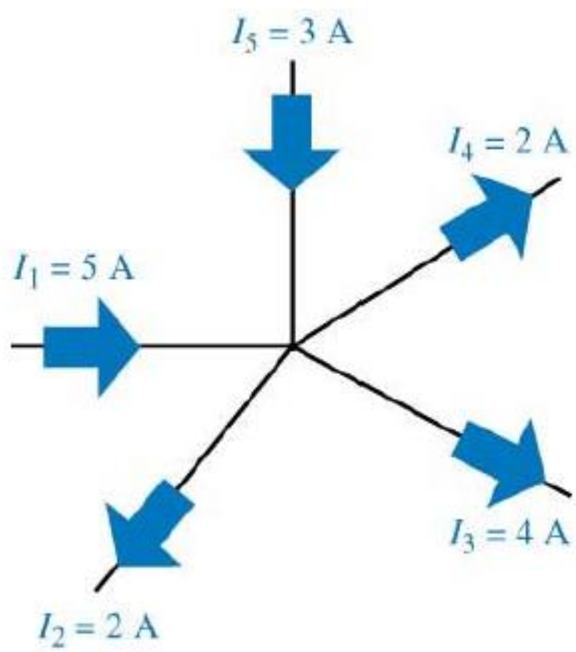
• "At any node (junction) in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node, or: The algebraic sum of currents in a network of conductors meeting at a point is zero".

•The sum of currents entering the junction are thus equal to the sum of currents leaving. This implies that the current is conserved (no loss of current).

Kirchhoff's Current Law (KCL) Kirchhoff's Current Law states that the algebraic sum of the currents entering and leaving a node is equal to zero

$$\sum I = 0$$

By convention, currents entering the node are positive, and those leaving a node are negative. For the picture at the right:

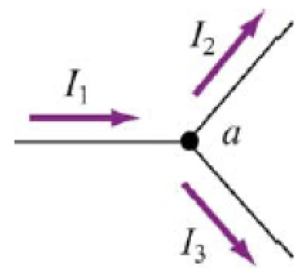


$$\sum_{n=1}^{N} I_n = I_1 + (-I_2) + (-I_3) + (-I_4) + I_5 = 0$$

KCL can also be expressed as “The sum of the currents entering a node is equal to the sum of the currents leaving a node

$$\sum I_{in} = \sum I_{out} \qquad I_1 + I_5 = I_2 + I_3 + I_4$$

- Example 2: If the currents exiting from junction "a" are to be of 2 amps each, what is the value for the current entering the junction?



Recall the junction rule for this case:

$$I_1 = I_2 + I_3$$

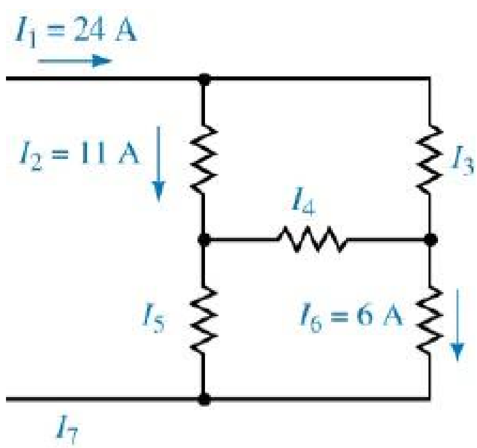
We know the following values:

$$I_2 = I_3 = 2\ amps$$

Then, we can solve for current entering the junction:

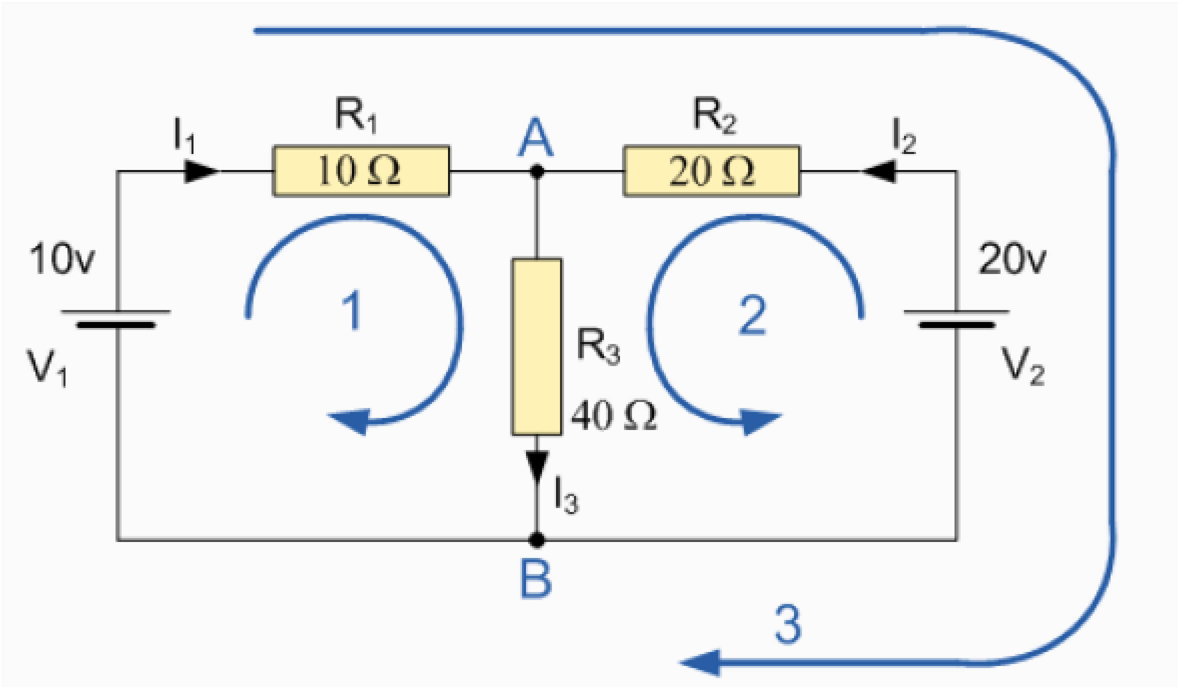
$$I_1 = 2 + 2 = 4\ amps$$

Example: Determine the unknown currents in the circuit shown below.

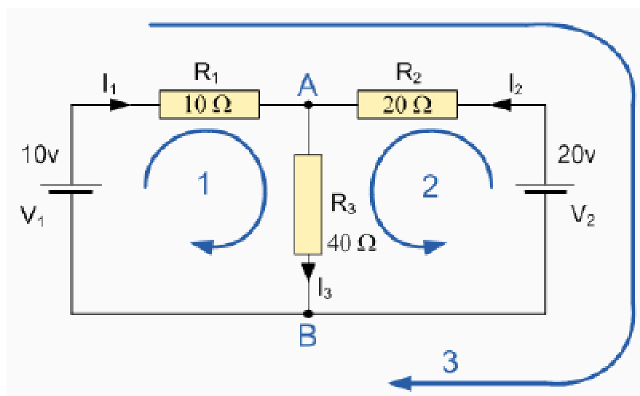


Solution:

Example 3: Determine the values of the the current flowing through each of the resistors.



- Example 3 (cont'd)
  The circuit has two nodes (at A and B). We have the choice of choosing only two of the three loops shown (blue). This is because only two of the loops are independent.



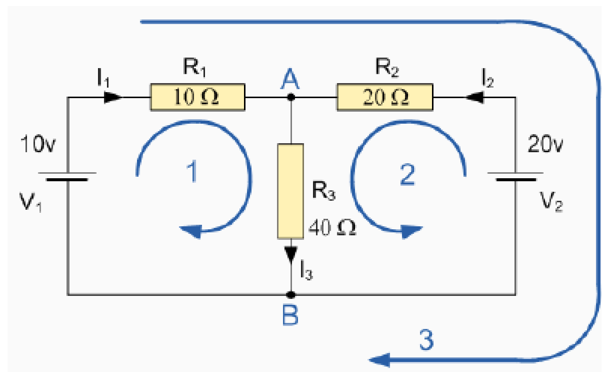
Node A $I_1 + I_2 = I_3$

Node B $I_3 = I_1 + I_2$

Loop 1 $10 - I_1 R_1 - I_3 R_3 = 0$

Loop 2 $20 - I_2 R_2 - I_3 R_3 = 0$

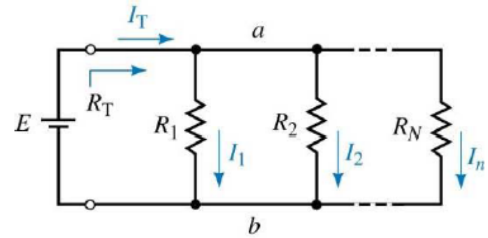
$$I_1 + I_2 = I_3$$
$$I_3 = I_1 + I_2$$
$$10 - I_1 R_1 - I_3 R_3 = 0$$
$$20 - I_2 R_2 - I_3 R_3 = 0$$



- By substitution, the answer can be shown to be I1=-0.143amps, and I2=0.429amps.

Resistors in Parallel Consider a circuit with 3 resistors in parallel (such as the circuit below, if $N = 3$).

$$I_T = I_1 + I_2 + I_3 \Rightarrow \frac{E}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$



Since the voltages across all the parallel elements in a circuit are the same ($E = V_1 = V_2 = V_3$), we have:

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3} \Rightarrow \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

This result can be generalized to provide the total resistance of any number of resistors in parallel:

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}}$$

Special Case: Two Resistors in Parallel For only two resistors connected in parallel, the equivalent resistance may be found by the product of the two values divided by the sum:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

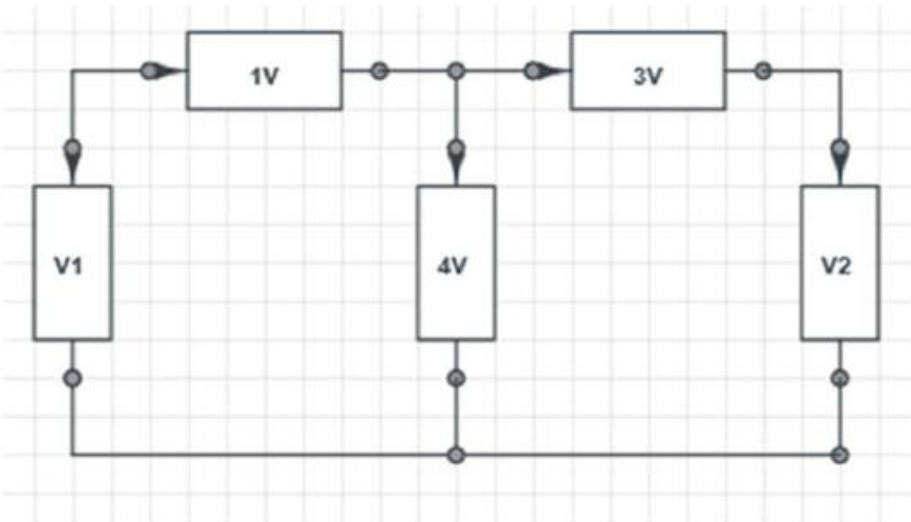
If you want to be cool, you should refer to this as the "product over the sum" formula. Your EE friends will really admire this.

Special Case: Equal Resistors in Parallel Total resistance of n equal resistors in parallel is equal to the resistor value divided by the number of resistors (n):

$$R_T = \frac{R}{n}$$

EXAMPLE

Find V_1 and V_2



Loop 1

$$-V_1 + 1 + 4 = 0 = V_1 = 5V$$

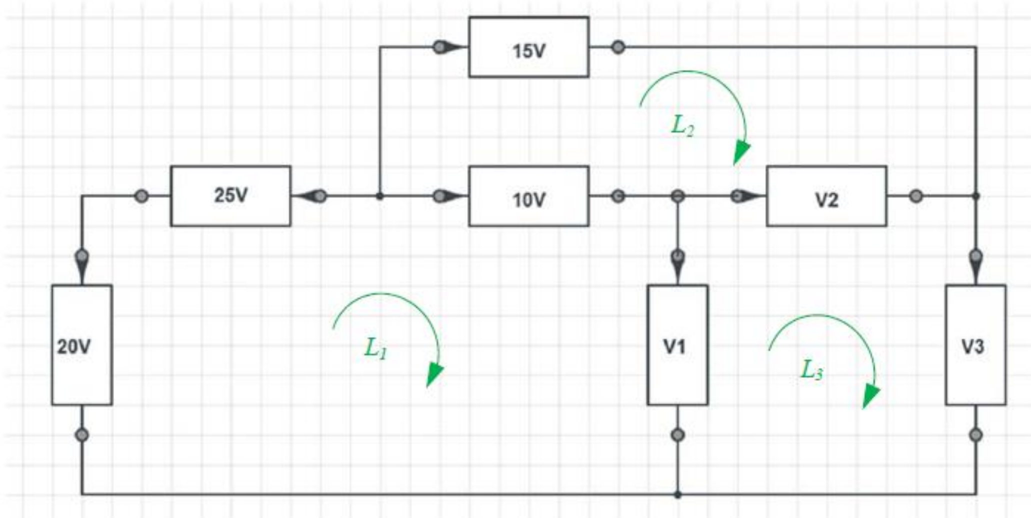
Loop 2

$$-4 + 3 + V_2 = 0 \Rightarrow V_2 = 1V$$

EXAMPLE

Find V_1, V_2, V_3

(note: the arrows are signifying the positive position of the box and the negative is at the end of the box)



Loop 1

$$-20 - 25 + 10 + V_1 = 0 \Rightarrow V_1 = 35V$$

Loop 2

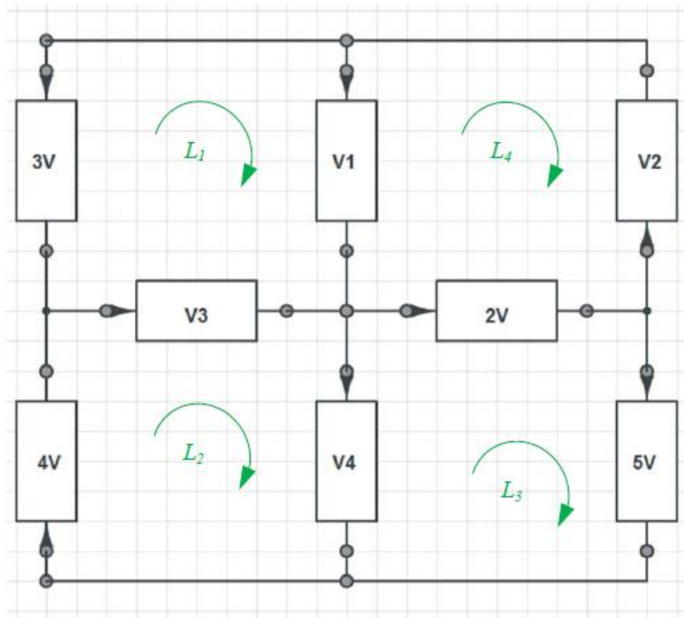
$$-10 + 15 - V_2 = 0 \Rightarrow V_2 = 5$$

Loop 3

$$-V_1 + V_2 + V_3 = 0 \Rightarrow -35 + 5 + V_3 = 0 \Rightarrow V_3 = 30V$$

Find V_1, V_2, V_3, V_4

(note: the arrows are signifying the positive position of the box and the negative is at the end of the box)



Loop 1

$$-V_4 + 2 + 5 = 0$$

$$V_4 = 7V$$

Loop 2

$$4 + V_3 + V_4 = 0$$

$$V_3 = -4 - 7$$

$$= -11V$$

Loop 3

$$-3 + V_1 - V_3 = 0$$

$$V_1 = V_3 + 3$$

$$= -11 + 3$$

$$= -8V$$

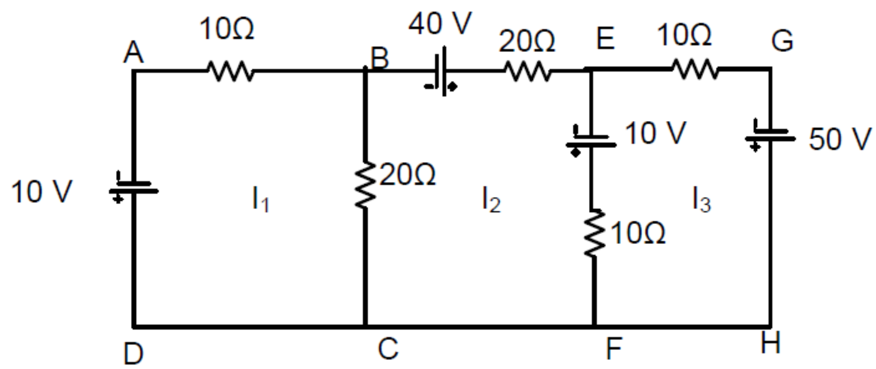
Loop 4

$$-V_1 - V_2 - 2 = 0$$

$$V_2 = -V_1 - 2$$

$$= 6V$$

Example: Find I_1, I_2 and I_3 in the network shown in Fig below using loop current method



Solution:- For mesh ABCDA,

$$-I_1\times 10-(I_1-I_2)\times 20-10=0$$
$$\Rightarrow\ 3I_1-2I_2=-1 \qquad (1)$$

For mesh BEFCB,

$$40-I_2\times 20+10-(I_2-I_3)\times 10-(I_2-I_1)\times 20=0$$
$$\Rightarrow\ 2I_1-5I_2+I_3=-5 \qquad (2)$$

For mesh EGHFE,

$$-10I_3+50-(I_3-I_2)\times 10-10=0$$
$$\Rightarrow\ I_2-2I_3=-4 \qquad (3)$$

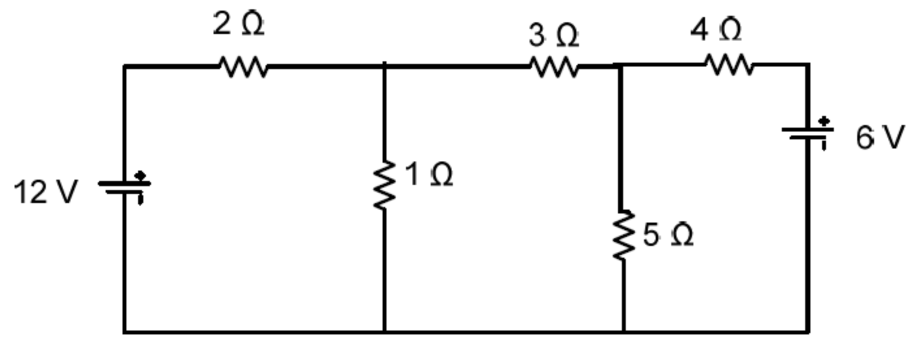
Equation (2) x 2 + Equation (3)

$$4I_1-9I_2=-14 \qquad (4)$$

Solving eq^n (1) & eq^n (4)

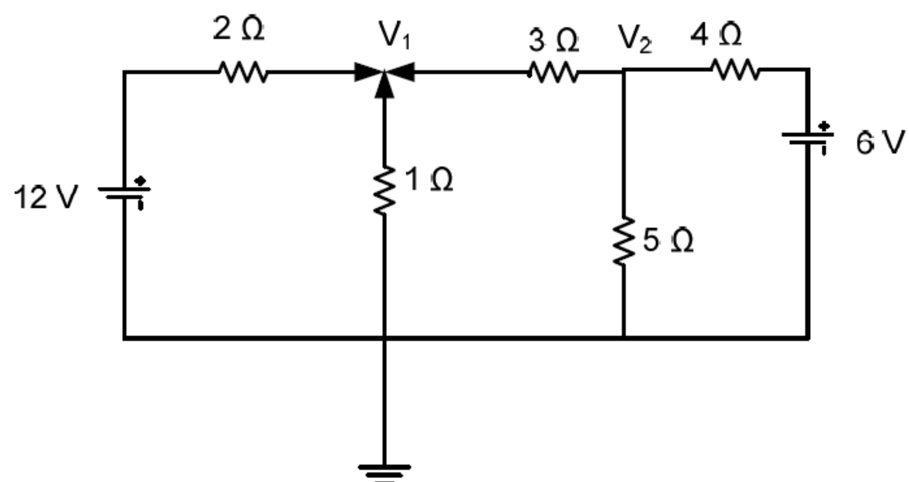
$I_1 = 1$ A, $I_2 = 2$ A, $I_3 = 3$ A

Example: - Use nodal analysis to find currents in the different branches of the circuit shown below.



Solution:-

Let V_1 and V_2 are the voltages of two nodes as shown in Fig below



Applying KCL to node-1, we get

$$\frac{12 - V_1}{2} + \frac{0 - V_1}{1} + \frac{V_2 - V_1}{3} = 0$$

$$\Rightarrow 36 - 3V_1 - 6V_1 + 2V_2 - 2V_1 = 0$$

$$\Rightarrow -11V_1 + 2V_2 = 36 \ldots\ldots\ldots\ldots\ldots(1)$$

Again applying KCL to node-2, we get:-

$$\frac{V_1 - V_2}{3} + \frac{0 - V_2}{5} + \frac{6 - V_2}{4} = 0$$

$$\Rightarrow 20V_1 - 47V_2 + 90 = 0$$

$$\Rightarrow 20V_1 - 47V_2 = -90 ....(2)$$

Solving Eq (1) and (2) we get $V_1 = 3.924$ Volt and $V_2 = 3.584$ volt

Current through 2 Ω resistance = $\frac{12\text{-}V_1}{2} = \frac{12\text{-}3.924}{2} = 4.038$ A

Current through 1 Ω resistance = $\frac{0\text{-}V_1}{1} = -3.924$ A

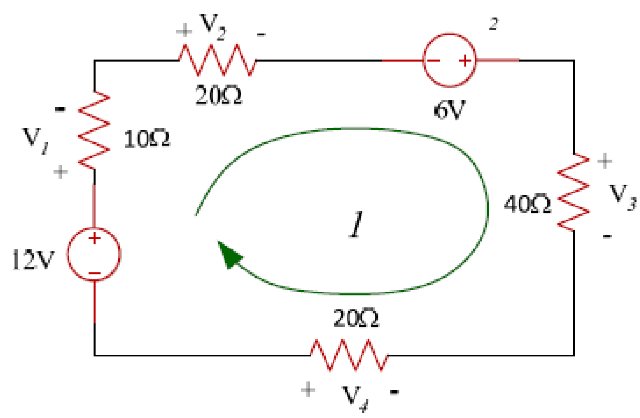
Current through 3 Ω resistance = $\frac{V_1\text{-}V_2}{3} = 0.1133$ A

Current through 5 Ω resistance = $\frac{0\text{-}V_2}{5} = -0.7168$ A

Current through 4 Ω resistance = $\frac{6\text{-}V_2}{4} = 0.604$ A

As currents through 1Ω and 5Ω are negative, so actually their directions are opposite to the assumptions.

Example.: Find the current I for the circuit shown.



KVL equations for voltages

$$v_1 + v_2 + v_3 - v_4 = 18$$

Using Ohm's Law

$v_1 = 10\Omega i$ $\quad v_2 = 20\Omega i$ $\quad v_3 = 40\Omega i$, $\quad v_4 = -20\Omega i$

Apply them into KVL eauation

$$10i + 20i + 40i + 20i = 18$$

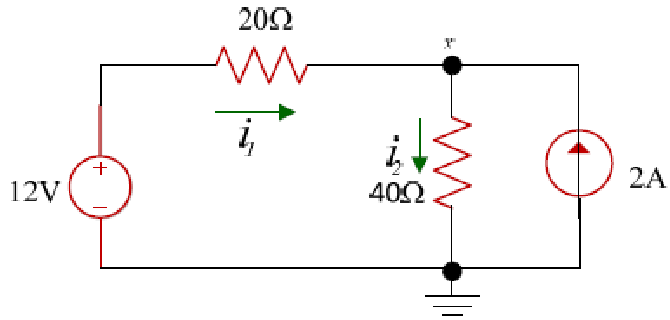
$$(90)i = 18$$

$$i = \frac{18}{90} = 0.2A$$

$V_1 = 10\Omega i = 10(0.2) = 2V$ $\qquad V_2 = 20\Omega i = 20(0.2) = 4V$

$V_3 = 40\Omega i = 40(0.2) = 8V$ $\qquad V_4 = 20\Omega i = 20(0.2) = 4V$

Example: Find the current through a 20 Ω resistor, and current through a 40 Ω resistor in the following circuit.



- Write KCL at node x

$$i_1 - 2_2 + 2A = 0$$

- Write v_x in the circuit using Ohm's Law

$$i_1 = \frac{12V - v_x}{20\Omega} \quad \text{and} \quad i_2 = \frac{v_x}{40\Omega}$$

- Apply them in to KCl equation

$$\frac{12V - v_x}{20\Omega} - \frac{v_x}{40\Omega} + 2A = 0$$

$$0.6 - 0.05v_x - 0.025v_x + 2A = 0$$

$$0.075v_x = 2.6A$$

$$v_x = 34,67V$$

$$i_1 = \frac{12V - v_x}{20\Omega} = \frac{12V - 34.67}{20\Omega} = -1.134A$$

$$i_2 = \frac{V_x}{40\Omega} = \frac{34.67}{40\Omega} = 0.867A$$

Example: For the circuit in Fig. shown, find voltages v_1 and v_2.

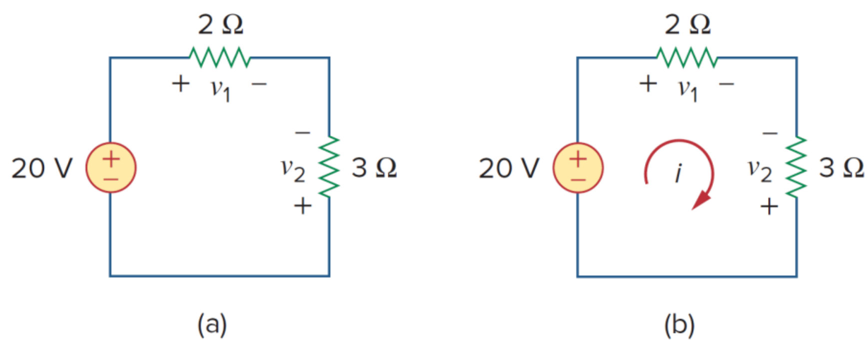


Figure 2.21
For Example 2.5.

Solution:

To find v_1 and v_2 we apply Ohm's law and Kirchhoff's voltage law. Assume that current i flows through the loop as shown in Fig. 2.21(b). From Ohm's law,

$$v_1 = 2i, \qquad v_2 = -3i \tag{2.5.1}$$

Applying KVL around the loop gives

$$-20 + v_1 - v_2 = 0 \tag{2.5.2}$$

Substituting Eq. (2.5.1) into Eq. (2.5.2), we obtain

$$-20 + 2i + 3i = 0 \qquad \text{or} \qquad 5i = 20 \qquad \Rightarrow \qquad i = 4 \text{ A}$$

Substituting i in Eq. (2.5.1) finally gives

$$v_1 = 8 \text{ V}, \qquad v_2 = -12 \text{ V}$$

Example: Find currents and voltages in the circuit shown in Fig

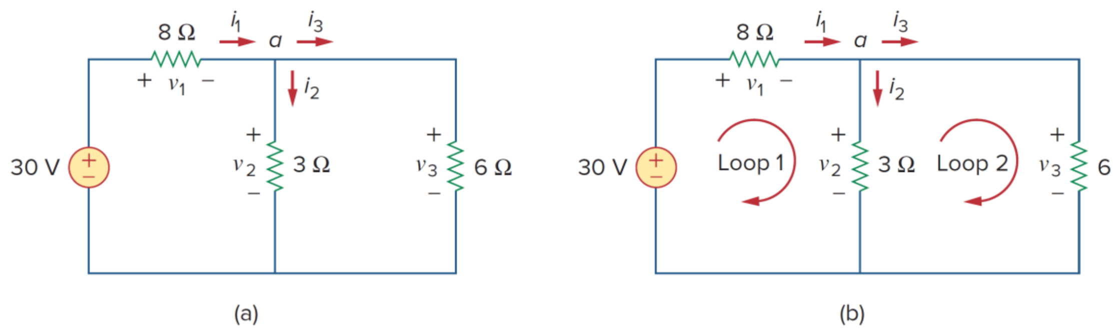


Figure 2.27
For Example 2.8.

Solution:
We apply Ohm's law and Kirchhoff's laws. By Ohm's law,

$$v_1 = 8i_1, \qquad v_2 = 3i_2, \qquad v_3 = 6i_3 \tag{2.8.1}$$

Since the voltage and current of each resistor are related by Ohm's law as shown, we are really looking for three things: (v_1, v_2, v_3) or (i_1, i_2, i_3). At node a, KCL gives

$$i_1 - i_2 - i_3 = 0 \tag{2.8.2}$$

Applying KVL to loop 1 as in Fig. 2.27(b),

$$-30 + v_1 + v_2 = 0$$

We express this in terms of i_1 and i_2 as in Eq. (2.8.1) to obtain

$$-30 + 8i_1 + 3i_2 = 0$$

or

$$i_1 = \frac{(30 - 3i_2)}{8} \tag{2.8.3}$$

Applying KVL to loop 2,

$$-v_2 + v_3 = 0 \qquad \Rightarrow \qquad v_3 = v_2 \qquad \textbf{(2.8.4)}$$

as expected since the two resistors are in parallel. We express v_1 and v_2 in terms of i_1 and i_2 as in Eq. (2.8.1). Equation (2.8.4) becomes

$$6i_3 = 3i_2 \qquad \Rightarrow \qquad i_3 = \frac{i_2}{2} \qquad \textbf{(2.8.5)}$$

Substituting Eqs. (2.8.3) and (2.8.5) into (2.8.2) gives

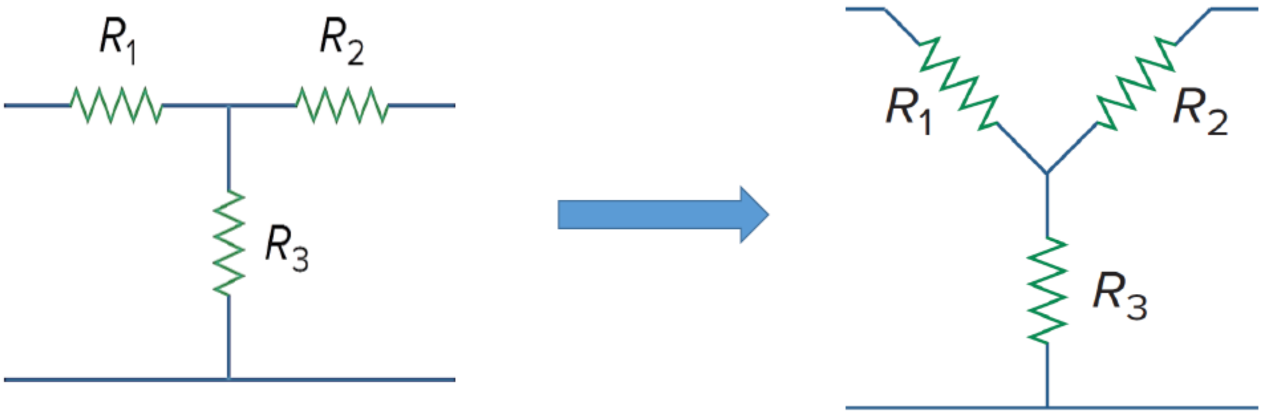
$$\frac{30 - 3i_2}{8} - i_2 - \frac{i_2}{2} = 0$$

or $i_2 = 2$ A. From the v alue of i_2, we no w use Eqs. (2.8.1) to (2.8.5) to obtain

$$i_1 = 3\text{ A}, \qquad i_3 = 1\text{ A}, \qquad v_1 = 24\text{ V}, \qquad v_2 = 6\text{ V}, \qquad v_3 = 6\text{ V}$$

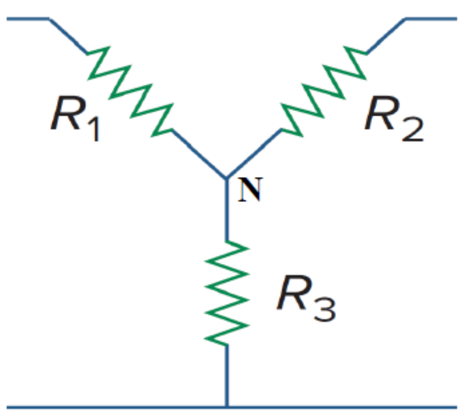
Star Delta Transformation for Resistive Networks

STAR CONNECTION Astar network is rearranged formof Tee(T) network



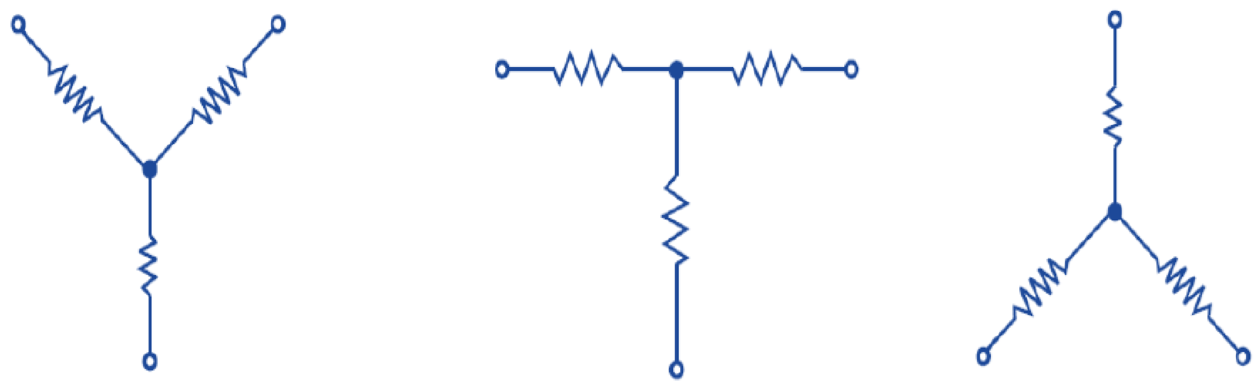
STAR/WYE(Y) CONNECTION

Three ends of resistors are connected in wye(Y) or star fashion. A common node point of star connection is known as neutral.



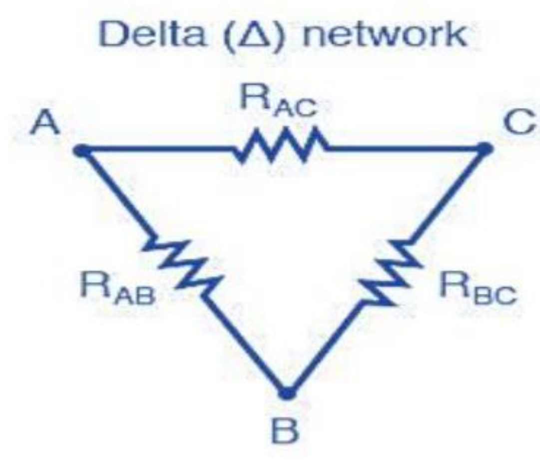
1- **STAR CONNECTION**

Three ways in which star connection may appear in a circuit.



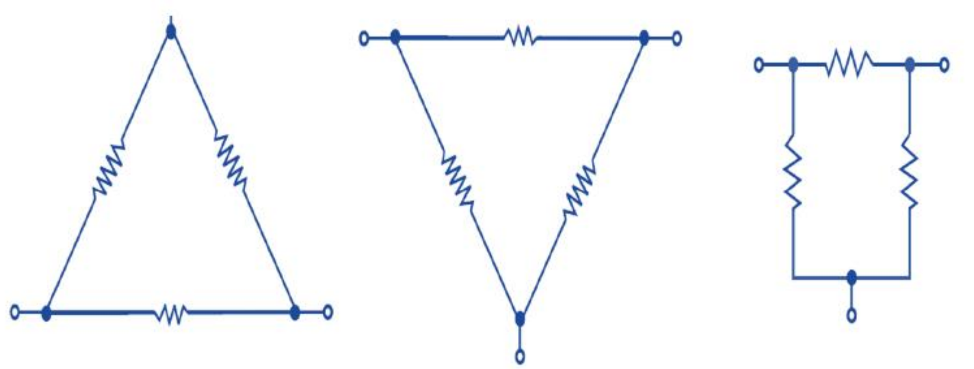
2- DELTA CONNECTION

When three resistors are connected in a fashion to form a closed mesh Δ, connection formed is known as Delta Connection.



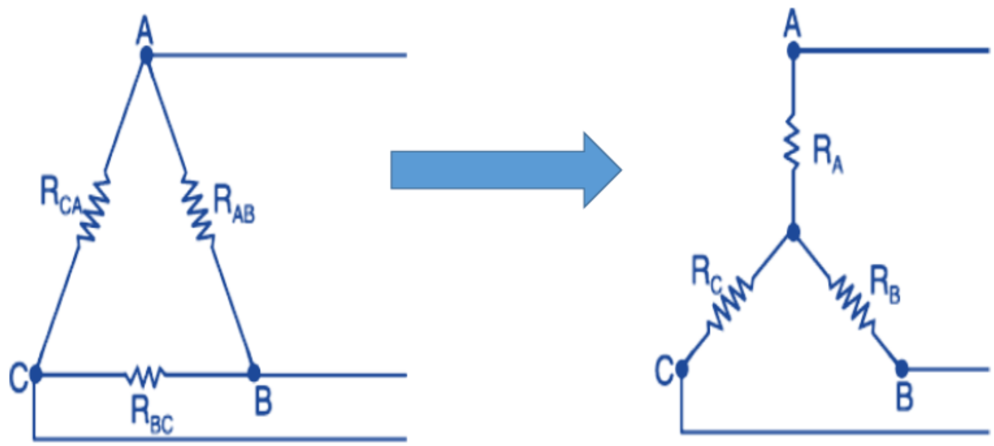
DELTACONNECTION

Three ways in which delta connection may appear in a circuit.



DELTA TO STAR TRANSFORMATION

Three resistors R_{AB}, R_{BC} *and* R_{CA} connected in delta form and its equivalent star connection is shown below.

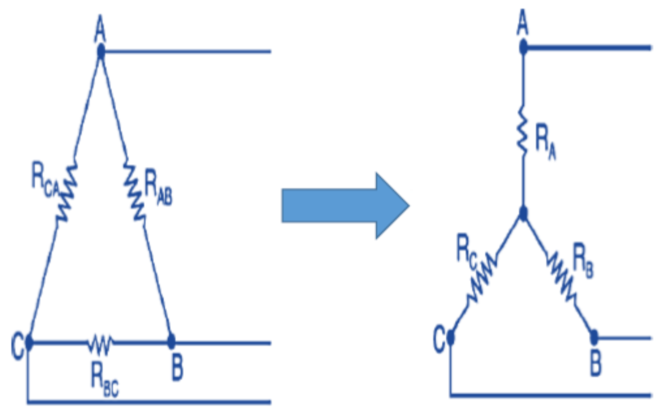


Delta and its equivalent Star

- Two arrangements shown are electrically equivalent.
- Resistance between A and B for star = Resistance between A and B for delta.
- Therefore,

$$R_A + R_B = R_{AB} \,||\, (R_{BC} + R_{CA}) \qquad (1)$$

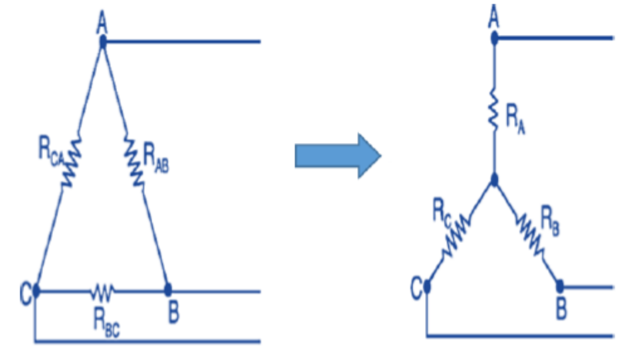
$$R_A + R_B = \frac{R_{AB}(R_{BC} + R_{CA})}{R_{AB} + R_{BC} + R_{CA}} \qquad (2)$$



- Similarly for resistance between two terminals B-C and C-A,

$$\Rightarrow R_B + R_C = \frac{R_{BC}(R_{CA} + R_{AB})}{R_{AB} + R_{BC} + R_{CA}} \quad (3)$$

$$\Rightarrow R_C + R_A = \frac{R_{CA}(R_{AB} + R_{BC})}{R_{AB} + R_{BC} + R_{CA}} \quad (4)$$

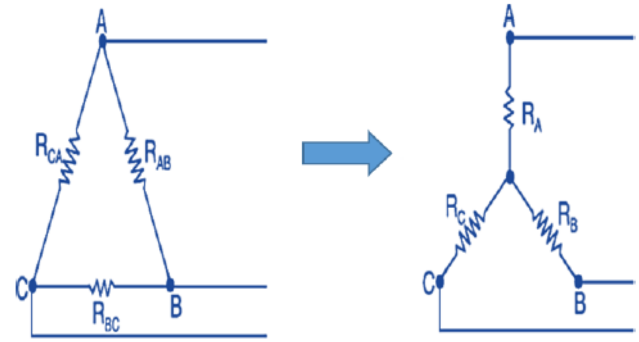


- The objective is to find R_A, R_B *and* R_C in terms of R_{AB}, R_{BC} *and* R_{CA} .
- Subtracting (3) from (2) and adding to (4) we obtain,

$$\Rightarrow R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \quad (5)$$

$$\Rightarrow R_B = \frac{R_{BC}R_{AB}}{R_{AB} + R_{BC} + R_{CA}} \quad (6)$$

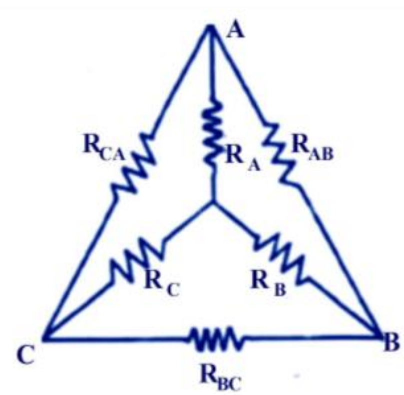
$$\Rightarrow R_C = \frac{R_{CA}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \quad (7)$$



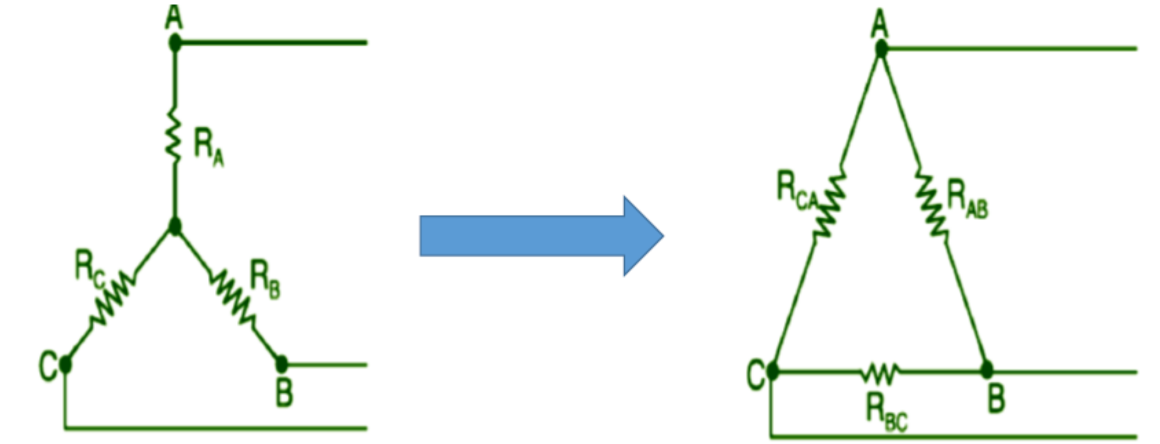
DELTA TO STAR TRANSFORMATION

- Easy way to remember delta to star transformation is,

$$\textbf{Any arm of star connection} = \frac{\textbf{Product of two adjacent arms of } \Delta}{\textbf{Sum of arms of } \Delta}$$



Three resistors R_A, R_B *and* R_C connected in star formation and its equivalent delta connection is shown below

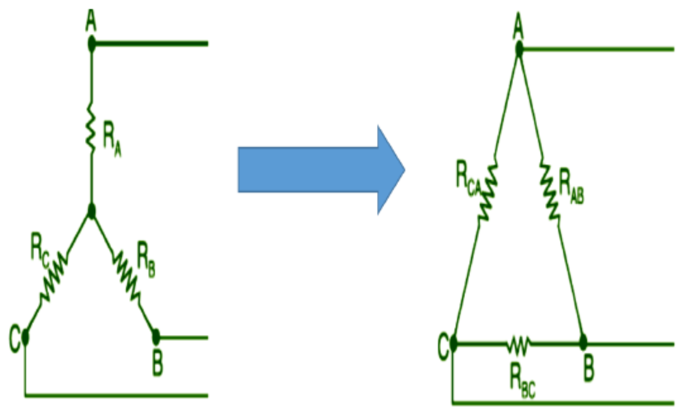


Star and its Equivalent Delta

- Dividing (5) by (6) we obtain,

$$\frac{R_A}{R_B} = \frac{R_{CA}}{R_{BC}} \qquad (8)$$

$$\Rightarrow R_{CA} = \frac{R_A R_{BC}}{R_B} \qquad (9)$$



- Dividing (5) by (7) we obtain,

$$\frac{R_A}{R_C} = \frac{R_{AB}}{R_{BC}} \qquad (10)$$

$$\Rightarrow R_{AB} = \frac{R_A R_{BC}}{R_C} \qquad (11)$$

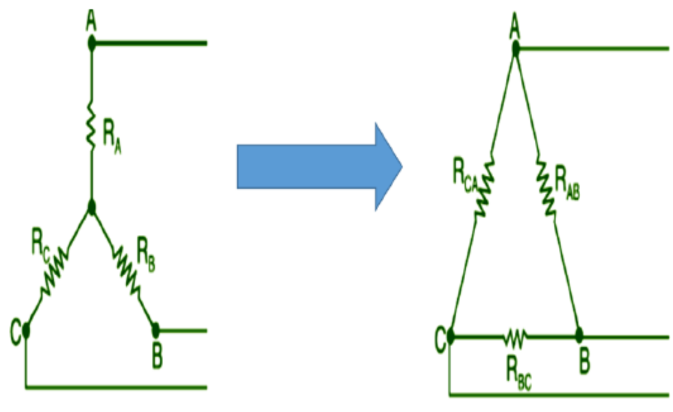
- Substituting (9) and (11) into (5),

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \qquad (12)$$

- Similarly,

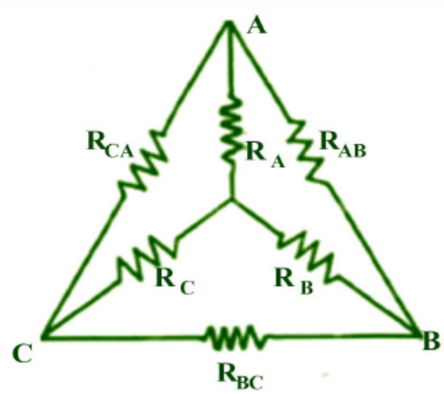
$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \qquad (13)$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B} \qquad (14)$$



- Easy way to remember star to delta transformation is,

Resistance between two terminals of Δ =
Sum of star resistances connected to those terminals + product of same two resistances divided by the third

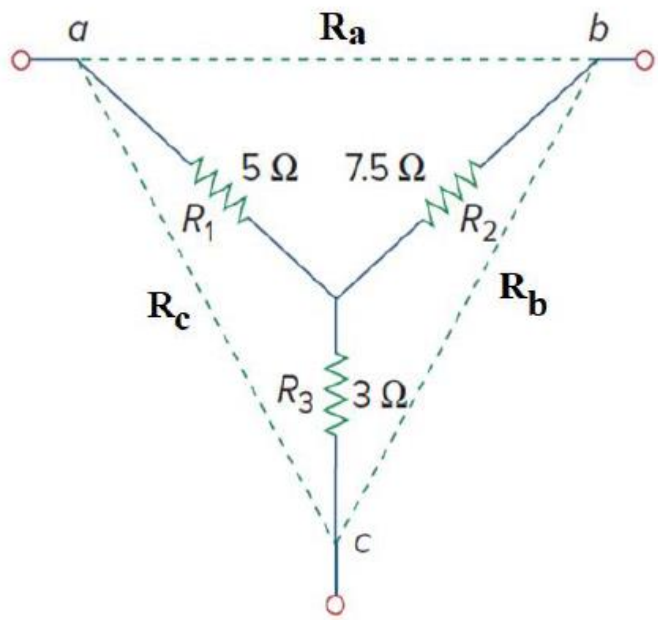


- If a star network has all resistances equal to R, its equivalent delta has all resistances equal to ?

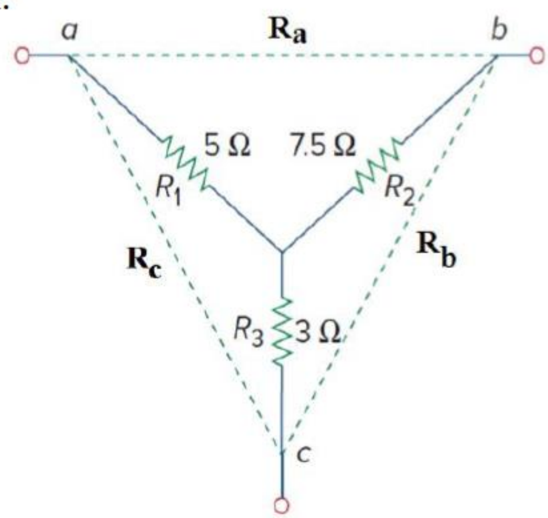
- If a delta network has all resistances equal to R, its equivalent star has all resistances equal to ?

Example :-

Q. Convert the Y network to an equivalent Δ network.



Soln:



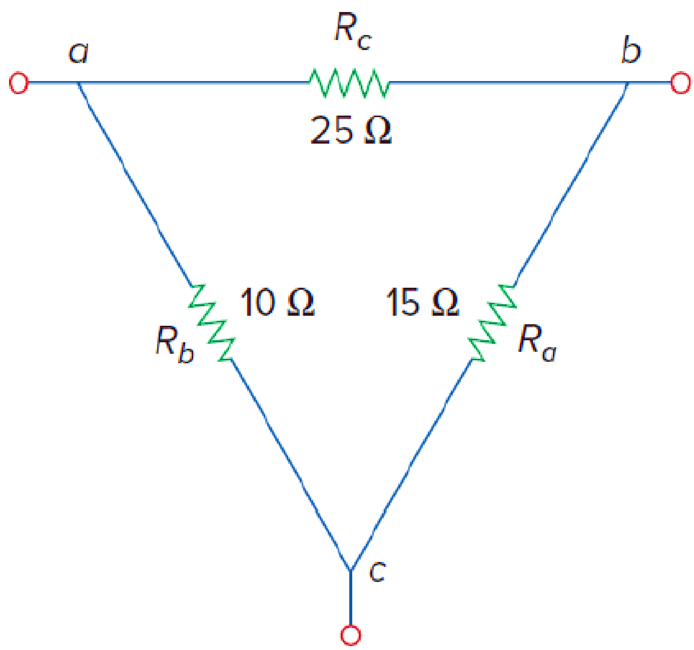
$$R_a = 7.5 + 5 + \frac{7.5 \times 5}{3} = 25 \; ohms$$

$$R_b = 7.5 + 3 + \frac{7.5 \times 3}{5} = 15 \; ohms$$

$$R_c = 5 + 3 + \frac{5 \times 3}{7.5} = 10 \; ohms$$

Example

Q. Convert the Δ network to an equivalent Y network.

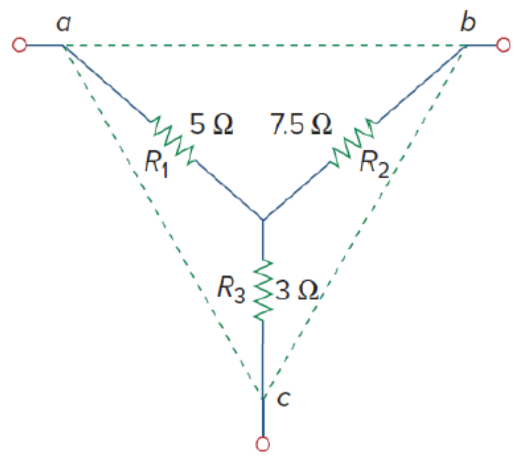


Soln:

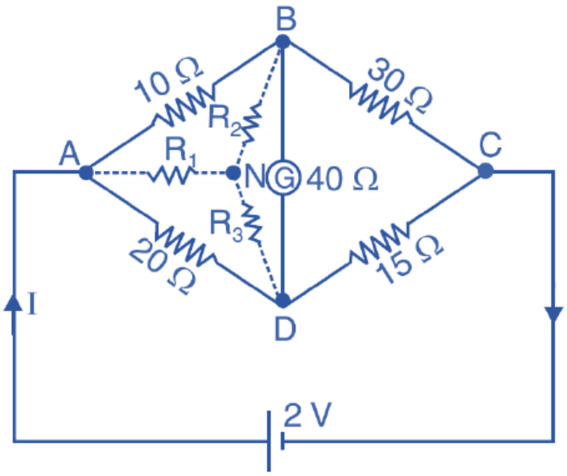
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = 5\ ohms$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ ohms$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3 ohms$$



Q. Using delta/star transformation, find equivalent resistance across AC.



Soln: Delta can be replaced by equivalent star-connected resistances,

$$R_1 = \frac{R_{AB}R_{DA}}{R_{AB}+R_{DA}+R_{BD}} = \frac{10\times 20}{10+40+20} = 2.86\ ohms$$

$$R_2 = \frac{R_{AB}R_{BD}}{R_{AB}+R_{DA}+R_{BD}} = \frac{10\times 40}{10+40+20} = 5.72\ ohms$$

$$R_3 = \frac{R_{DA}R_{BD}}{R_{AB}+R_{DA}+R_{BD}} = \frac{10\times 40}{10+40+20} = 11.4\ ohms$$

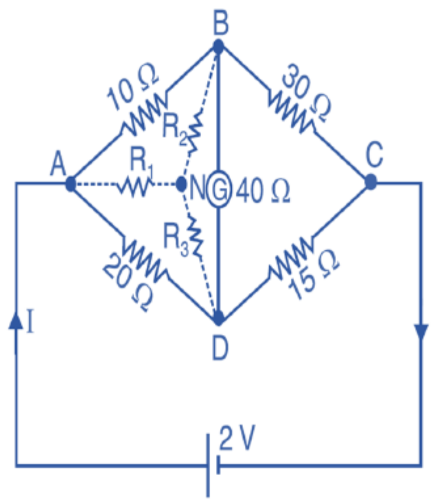
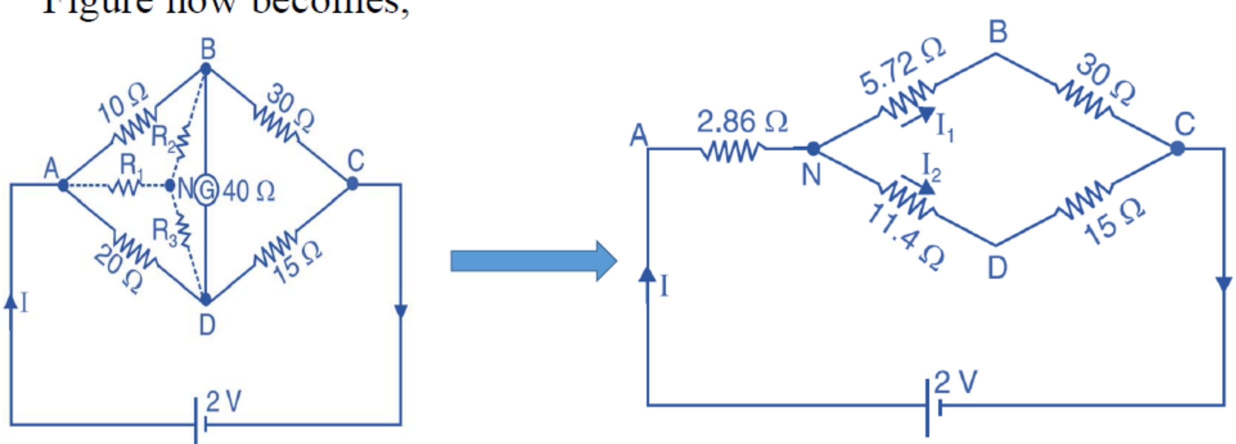
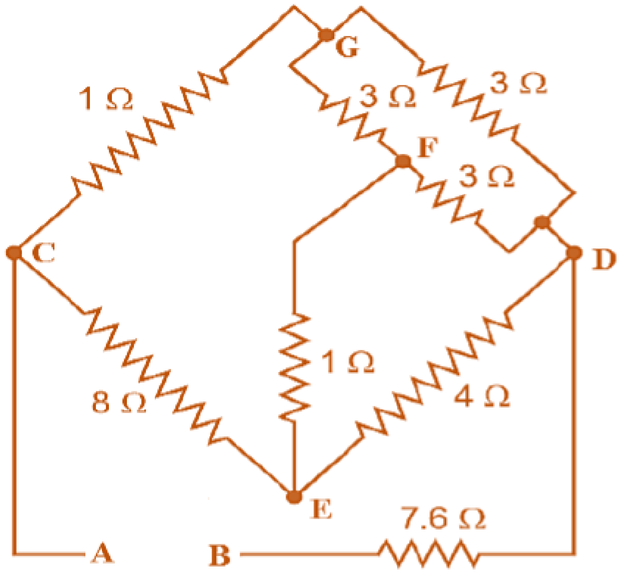


Figure now becomes,

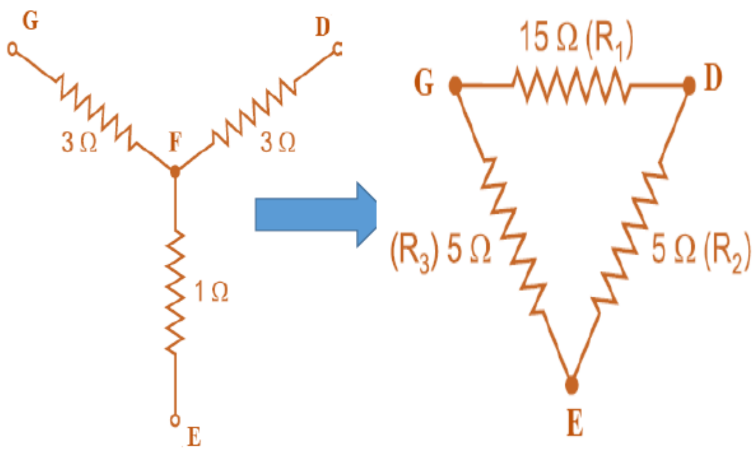


$$R_{AC} = 2.86 + \frac{(30+5.72)\times(15+11.4)}{(30+5.72)+(15+11.4)} = 18.04\ ohms$$

Q. Calculate equivalent resistance across terminals A and B.



Soln: Converting inner STAR (3 ohms, 3 ohms and 1 ohms) into Delta.

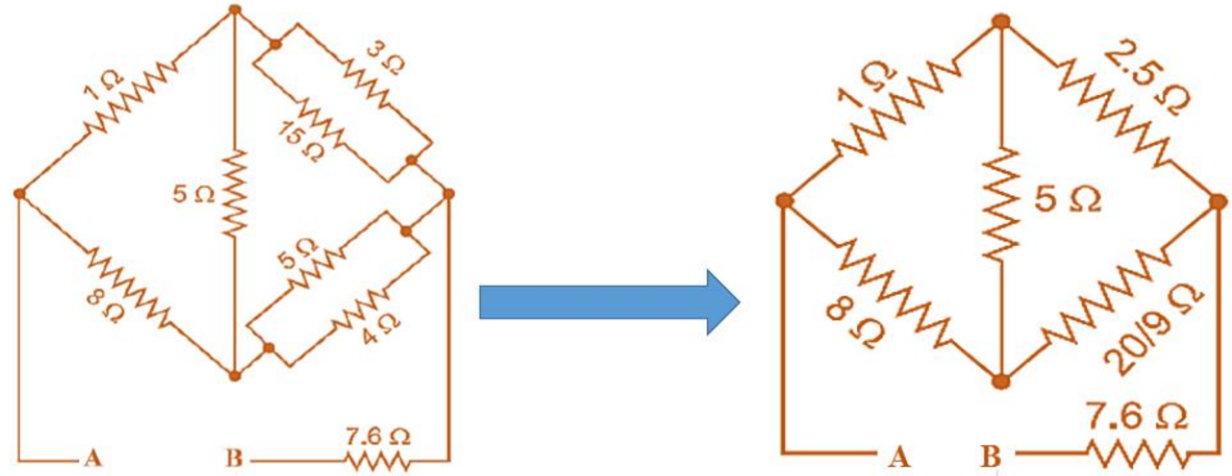


$$R_1 = 3 + 3 + \frac{3 \times 3}{1} = 15 \; ohms$$

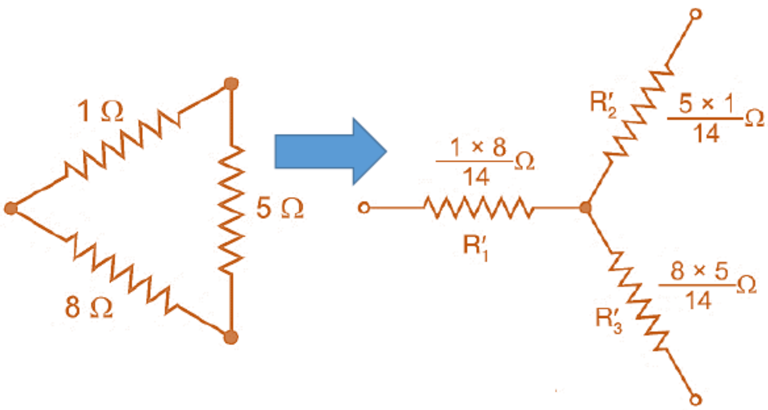
$$R_2 = 3 + 1 + \frac{3 \times 1}{3} = 5 \; ohms$$

$$R_3 = 1 + 3 + \frac{1 \times 3}{3} = 5 \; ohms$$

Circuit now becomes,



Delta-connected resistances 1 Ω, 5 Ω and 8 are converted in star,

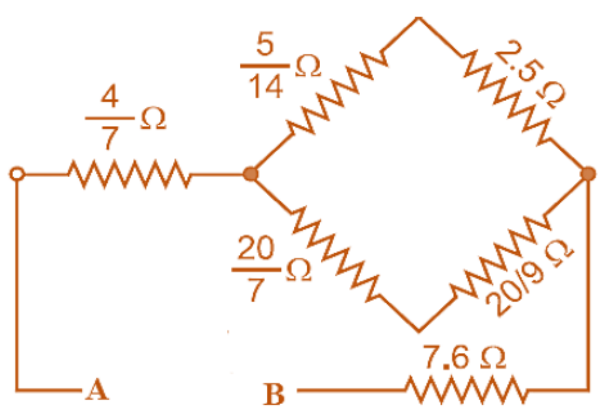


$$R_1' = \frac{1\times 8}{1+5+8} = \frac{4}{7}\,ohms$$

$$R_2' = \frac{5\times 1}{1+5+8} = \frac{5}{14}\,ohms$$

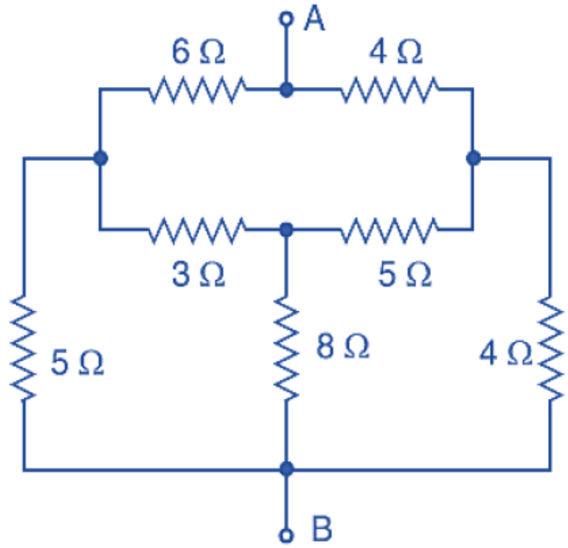
$$R_3' = \frac{8\times 5}{1+5+8} = \frac{20}{7}\,ohms$$

Circuit now becomes,



$$R_{AB} = \frac{4}{7} + \left[\left(\frac{5}{14}+2.5\right) \parallel \left(\frac{20}{7}+\frac{20}{9}\right)\right] + 7.6 = 10\ ohms$$

Q. Calculate equivalent resistance across terminals A and B.



Nodal analysis and Mesh analysis

Steps of Mesh Analysis:-

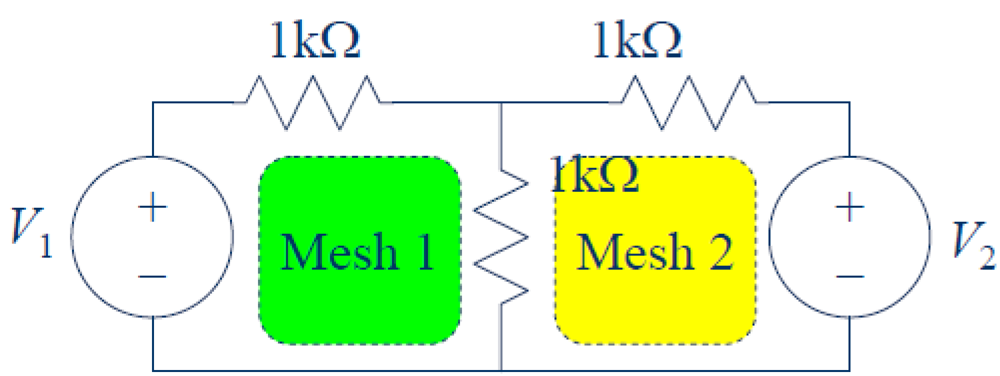
1.Identify meshes.

2.Assign a current to each mesh.

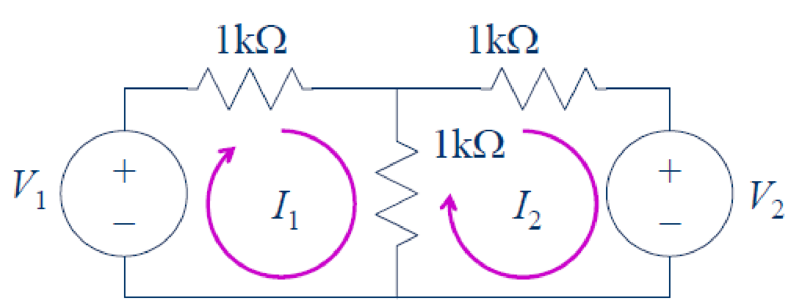
3.Apply KVL around each loop to get an equation in terms of the loop currents.

4.Solve the resulting system of linear equations.

Identifying the Meshes.



2- Assigning Mesh Currents.

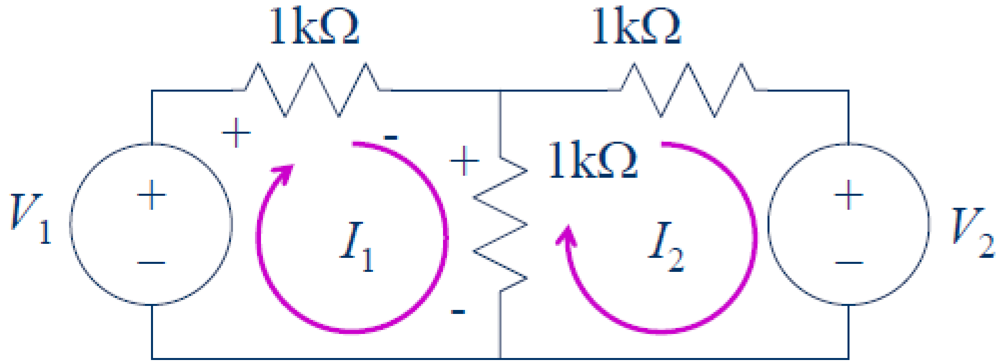


3- Apply KVL around each loop to get an equation in terms of the loop currents

3-1 Voltages from Mesh Currents.

$$V_R = I_1 R \qquad\qquad V_R = (I_1 - I_2)\, R$$

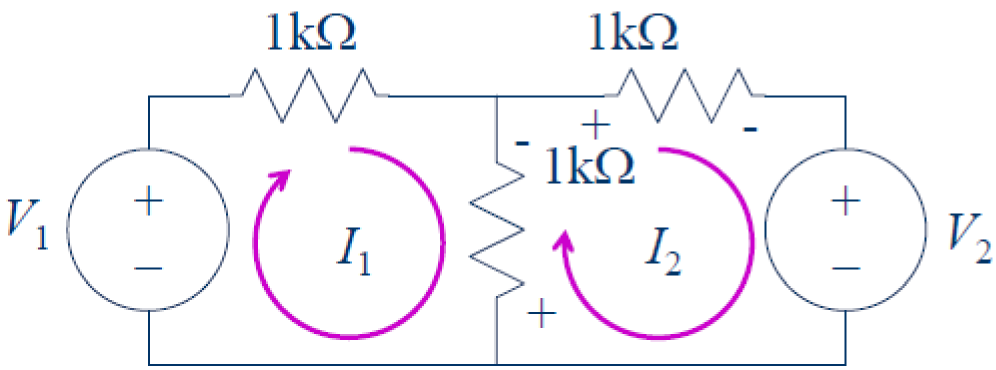
3-2 KVL Around Mesh 1.



$$-V_1 + I_1\, 1k\Omega + (I_1 - I_2)\, 1k\Omega = 0$$

$$I_1\, 1k\Omega + (I_1 - I_2)\, 1k\Omega = V_1$$

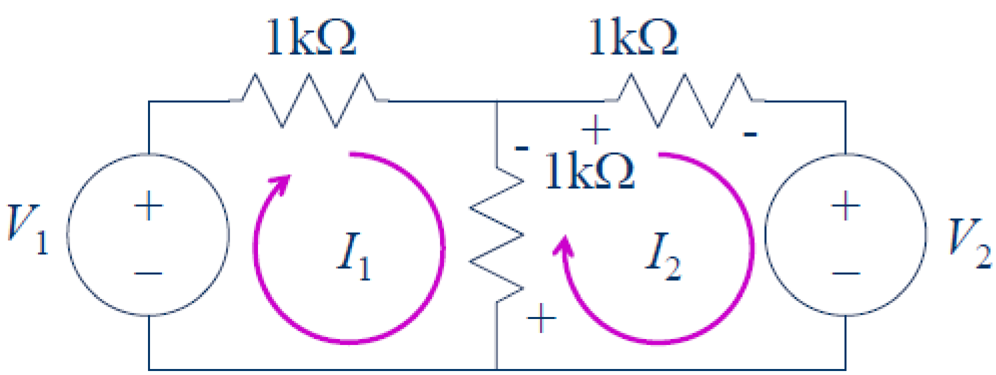
3-3 KVL Around Mesh 2.



$$(I_2 - I_1)\ 1k\Omega + I_2\ 1k\Omega + V_2 = 0$$

$$(I_2 - I_1)\ 1k\Omega + I_2\ 1k\Omega = -V_2$$

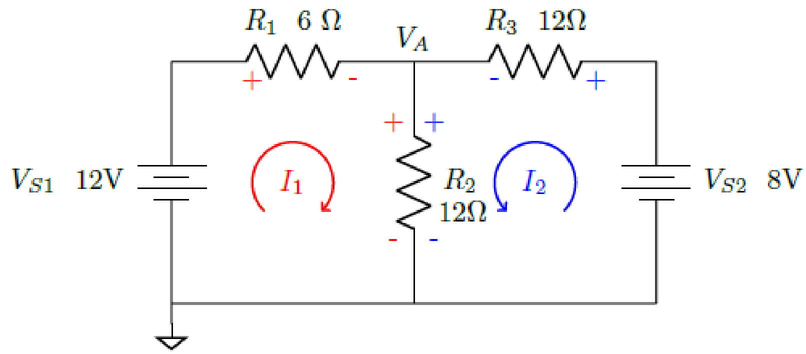
4- Solve the resulting system of linear equations.



$$(I_2 - I_1)\ 1k\Omega + I_2\ 1k\Omega + V_2 = 0$$

$$(I_2 - I_1)\ 1k\Omega + I_2\ 1k\Omega = -V_2$$

Example: Find VA



Solution

Step 1 was already taken care of above. Notice that here the first mesh current is owing CW and the second is owing counter-clockwise (CCW). In this case the currents through R2 are owing in the same direction and therefore the polarities are identical. This will result in VR2 having the same sign regardless of which mesh is being analyzed. The polarities are also already marked for Step 2. The KVL equations are

$$V_{S1} - V_{R1} - V_{R2} = 0$$
$$V_{S2} - V_{R2} - V_{R3} = 0$$

Using Ohm's Law for Step 3 give us:

$$V_{S1} - I_1 R_1 - (I_1 + I_2)R_2 = 0$$
$$V_{S2} - (I_1 + I_2)R_2 - I_2 R_3 = 0$$

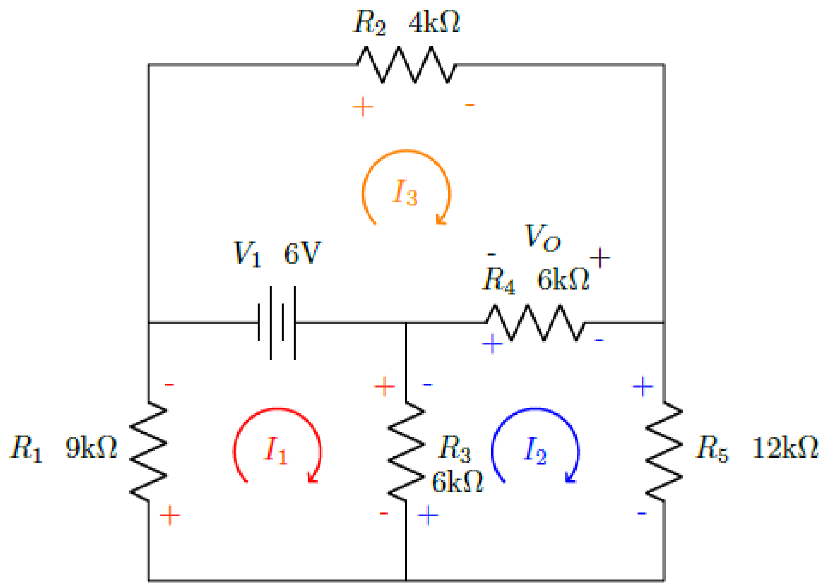
Distributing and grouping terms as prescribed in Step 4 results in

$$(R_1 + R_2)I_1 + R_2 I_2 = V_{S1}$$
$$R_2 I_1 + (R_2 + R_3)I_2 = V_{S2}$$

and after substituting values

$$18\Omega I_1 + 12\Omega I_2 = 12\ \text{V}$$
$$12\Omega I_1 + 24\Omega I_2 = 8\ \text{V}$$

Example: Find V_O using mesh analysis.



Solution

KVL Equations:

$$-V_{R1} + V_1 - V_{R3} = 0$$
$$-V_{R3} - V_{R4} - V_{R5} = 0$$
$$-V_{R2} - V_{R4} - V_1 = 0$$

Substituting with Ohm's Law

$$-I_1 R_1 + V_1 - (I_1 - I_2)R_3 = 0$$
$$-(I_2 - I_1)R_3 - (I_2 - I_3)R_4 - I_2 R_5 = 0$$
$$-I_3 R_2 - (I_3 - I_2)R_4 - V_1 = 0$$

Grouping Like-terms

$$(-R_1 - R_3)I_1 + R_3 I_2 = -V_1$$
$$R_3 I_1 + (-R_3 - R_4 - R_5)I_2 + R_4 I_3 = 0$$
$$R_4 I_2 + (-R_2 - R_4)I_3 = V_1$$

Substituting and Solving

$$-15k\Omega I_1 + 6k\Omega I_2 = -6\text{ V}$$
$$6k\Omega I_1 - 24k\Omega I_2 + 6k\Omega I_3 = 0$$
$$6k\Omega I_2 - 10k\Omega I_3 = 6\text{ V}$$

Solve using matrices:

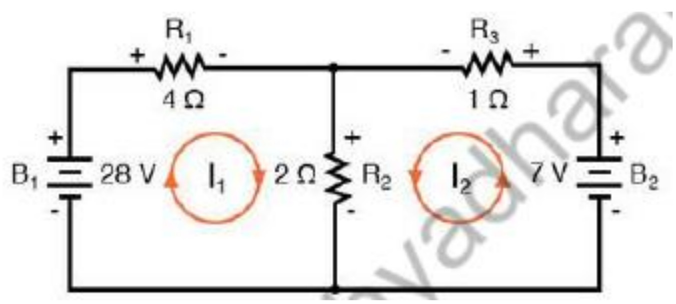
$$\begin{bmatrix} -15\text{ k}\Omega & 6\text{ k}\Omega & 0\ \Omega \\ 6\text{ k}\Omega & -24\text{ k}\Omega & 6\text{ k}\Omega \\ 0\ \Omega & 6\text{ k}\Omega & -10\text{ k}\Omega \end{bmatrix}^{-1} \begin{bmatrix} -6\text{ V} \\ 0\text{ V} \\ 6\text{ V} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 373.33\ \mu\text{A} \\ -66.67\ \mu\text{A} \\ -640.00\ \mu\text{A} \end{bmatrix}$$

Finally solving for V_O

$$V_O = (I_3 - I_2)R_4 = (-640.00\ \mu\text{A} + 66.67\ \mu\text{A})6\text{ k}\Omega = -3.44\text{V}$$

Example /

Using mesh analysis, obtain the current through the various components

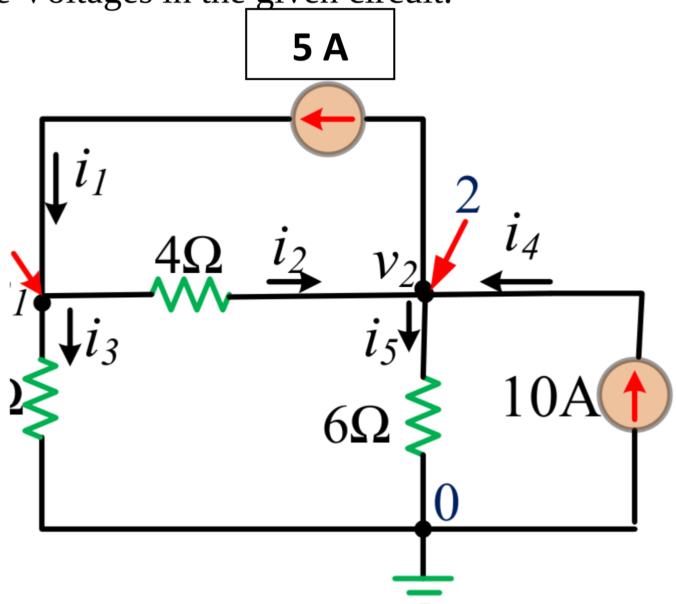


$$28 = 4\,i_1 + 2(i_1 + i_2)$$
$$28 = 6\,i_1 + 2i_2$$

$$7 = i_2 + 2(i_1 + i_2)$$
$$7 = 2i_1 + 3i_2$$

$$i_1 = 5\ A$$
$$i_2 = -1\ A$$

Example / Calculate Node Voltages in the given circuit.



KCL at Node 1:

$$i_1 = i_2 + i_3$$

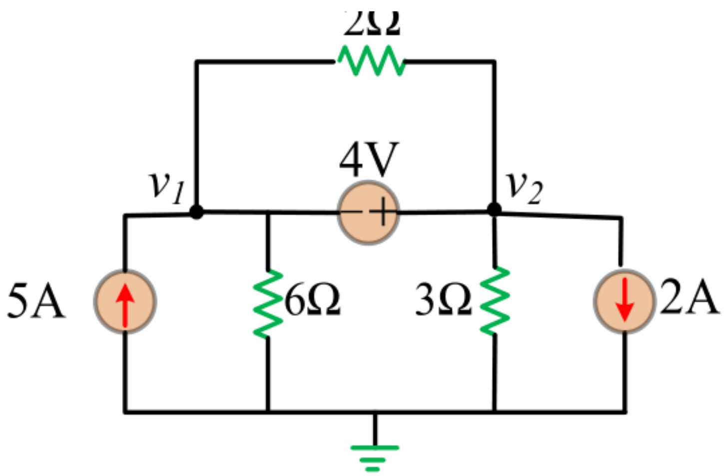
KCL at Node 2:

$$i_2 + i_4 = i_1 + i_5$$

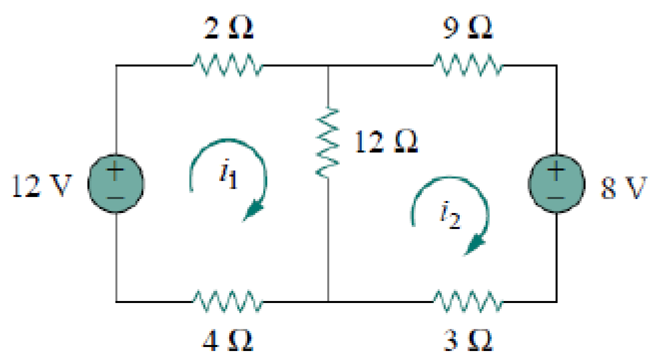
Ohm's Law to KCL equation at Node 1:

$$i_1 = i_2 + i_3 \Rightarrow 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2} \Rightarrow 3v_1 - v_2 = 20 \quad ...(1)$$

Q1/ Calculate Node Voltages in the given circuit.



Q2/Calculate the mesh currents i1 and i2 in the circuit shown



Q3/ Use mesh analysis to determine i1, i2, and i3 in the circuit shown